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# WALL HEATING BY A SUPERSONIC GAS FLOW

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**Abstract**—This work deals with the problem of wall heating by a supersonic gas flow. The solution of the problem is achieved by a modified form of the Chapman–Rubesin method, i.e. it is assumed that the temperature distribution along the wall may be expressed by the polynome:

$$T_w \cong T_0 + Ax^* + B(x^*)^2.$$

The results of the theoretical investigations are compared with observations of the flight of a V-2 rocket by Fischer and Norris.

**Résumé**—Ce travail traite le problème de l'échauffement des parois par un écoulement de gaz supersonique. La solution de ce problème est fournie par une forme modifiée de la méthode de Chapman–Rubesin; par exemple, on suppose que la distribution des températures le long de la paroi peut s'exprimer par le polynôme:

$$T_w \cong T_0 + Ax^* + B(x^*)^2.$$

Les résultats des recherches théoriques sont comparés aux observations faites, lors de vols de fusées V-2, par Fischer et Norris.

**Zusammenfassung**—In dieser Arbeit wird das Problem der Wandheizung durch die Überschallströmung eines Gases behandelt. Hierzu wird ein abgeändertes Verfahren nach Chapman und Rubesin angewandt, wonach für die Temperaturverteilung längs der Wand der Ausdruck

$$T_w \cong T_0 + Ax^* + B(x^*)^2$$

angenommen wird. Die Ergebnisse der theoretischen Untersuchung werden mit den Beobachtungen von Fischer und Norris an einer V2-Rakete verglichen.

**Аннотация**—В работе обсуждается задача о нагревании стенки сверхзвуковым потоком газа. Решение задачи проводится видоизмененным методом Чепмана-Рубезина, т.е. полагается, что распределение температур вдоль стенки можно представить полиномом

$$T_w \cong T_0 + Ax^* + Б(x^*)^2.$$

Результаты теоретического исследования сопоставляются с данными наблюдений Фишера и Норриса за полётом ракеты «V-2».

## 1. STATEMENT OF THE PROBLEM

THE problem of wall heating by a supersonic gas flow consists in the combined consideration of two particular problems:

(a) viscous gas heating by a shock wave;
(b) wall heating by heat transfer from the flowing gas.

The temperature distribution at the skin of the wall, in the direction of the flow, is the main unknown quantity in these two problems. Chapman and Rubesin [1] showed that their relationship may be expressed as a polynome:

$$T_w \cong T_0 + Ax^* + B(x^*)^2. \tag{1}$$

Here $T_0$ is the known temperature at the nose of the wall which is, generally speaking, variable in time, and

$$x^* = \frac{x}{L};$$

where $L$ is the length of the wall, $x$ is the distance from the nose to any point on the wall along the flow. The coefficients $A$ and $B$ depending on time are to be determined.

The following notions can serve as the basis of the physical interpretation of such a representation:

1. The temperature at the nose of the wall must approach that of the disturbed side of the shock wave:

$$T_0 \cong T_2 \tag{2}$$

$$T_2 \equiv T_1 + \frac{2\gamma}{(\gamma-1)^2}\frac{c_1}{c_p}M_{in}^2\left(1-\frac{1}{1-M_{in}^2}\right) \times \dagger \times \left(1+\frac{1}{\gamma}\frac{1}{M_{in}^2}\right) \tag{2'}$$

where $M_{in} \equiv U_{in}/c_1$ is the Mach parameter and the known function of time, $U_{in}$ is the component of undisturbed velocity $U_1$, normal to the shock wave, $T_1$ is the gas temperature at the undisturbed side of the shock wave.

2. At a great distance from the nose the wall temperature is equalized and at $x^* \to 1$‡ it ceases to vary:

$$\frac{\partial T_w}{\partial x^*} \cong 0.$$

Then by differentiating equation (1) we will get

$$A \cong -2B.$$

Now equation (1) can be written in the following form:

$$T_w \cong T_2 - B(t)x^*(2-x^*). \tag{1'}$$

Both the statement of the problem and its solution can be simplified.

The gas has a small thermal inertia because of the low volumetric heat capacity of the gas in comparison with the heat capacity of the wall streamlined by it. For equal periods of time the wall and the gas will exchange equal amounts of heat, i.e., the gas will deliver heat and the wall will take it up. Heat loss by the gas is compensated by the convective heat supply and is simultaneously accompanied by a cooling of the gas.

If the gas is at rest then loss of heat by it only results in its being cooled. At high velocities, loss of heat by the gas to the wall is only compensated by convection as a result of its small thermal inertia (under this condition a cooling of the gas will hardly occur). Owing to this a change in thermal state of the gas during small periods of time may be taken as quasi-stationary.

The wall heating, in contrast to the heating of the gas, is unsteady particularly during the first moments of the flow.§

The temperature field of the gas flowing along the wall, both at low and at high velocities, can be expressed formally through the surface temperature of the wall.

When the gas flowing along the wall is dense and the surface temperature is expressed as a power function in the direction of the flow, the solution of the problem can be found by a modified form of the Chapman–Rubesin method [1].||

If the gas is rarefied, the surface temperature being similarly represented as a power function, the solution of the problem can be found by the method we suggested in [2].

The problem of heating the wall (Fig. 1) is a problem in thermal conductivity:

$$c_w\rho_w\frac{\partial T_w}{\partial t} = \frac{\partial}{\partial y'}\left(\Lambda_w\frac{\partial T_w}{\partial y'}\right) + \frac{\partial}{\partial x'}\left(\Lambda_w\frac{\partial T_w}{\partial x'}\right)$$

where $c_w$, $\rho_w$, $\Lambda_w$ are the specific heat, the density and the thermal conductivity of the wall, respectively.

When the gas heated by the shock wave flows along the outer surface of the wall:

$$\Lambda_w\frac{\partial T_w}{\partial y'}\bigg|_{y'=0} = q_0$$

where $q_0$ is the heat flow from the gas to the wall.

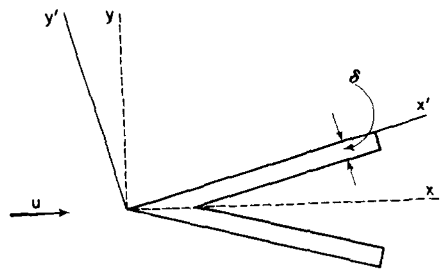


FIG. 1. The calculation scheme.

† The front part of the body is assumed to be a wedge with half open angle $\alpha$.

‡ For a great wall length $L$.

§ Such a scheme of the wall heating by gas was suggested by corr. m. Ac. Sc. U.S.S.R. I. A. Kibel, 1939.

|| The modification of the method is given in the next section.

In case the gas flowing along the wall is dense:

$$q_0 = \Lambda \frac{\partial T}{\partial y'}\bigg|_{y'=0}$$

$\Lambda$, $T$ are the thermal conductivity and the gas temperature, respectively.

In case rarefied gas flows along the wall:

$$q_0 = a\,(T - T_w)$$

where $a$ is the coefficient of heat transfer expressed by the temperature jump.

Heat transfer occurs on the internal side of the wall with the medium being inside the cavity of the body. This heat transfer is smaller than that along the outer side and can be neglected in comparison with the latter. For $y = -\delta$, it may be taken that at the inner side of the wall:

$$\frac{\partial T_w}{\partial y'}\bigg|_{y=-\delta} \cong 0.$$

The temperature at the nose of the wall can be assumed equal to the temperature of the disturbed side of the shock wave:

$$T_w|_{x^*=0} = T_{sh} \cong T_2.$$

Let us average the left- and the right-hand sides of the equation of heat conduction through the thickness of the wall:

$$c_w \rho_w\, \delta \frac{\partial \bar{T}_w}{\partial t} = \Lambda_w \frac{\partial T_w}{\partial y'} + \delta \frac{\partial}{\partial x'}\left(\Lambda_w \frac{\partial \bar{T}_w}{\partial x'}\right).$$

Here

$$\bar{T}_w \equiv \frac{1}{\delta}\int_0^\delta T_w\, \mathrm{d}y'.$$

If we neglect thermal conductivity along the wall we will have:

$$c_w \rho_w\, \delta \frac{\partial \bar{T}_w}{\partial t} \cong \Lambda \frac{\partial T}{\partial y'}. \qquad (3')$$

The equation obtained (3′) together with the equation of heat supply to the wall by the gas is the rated one.

As was shown above, the solution of this equation can be expressed through the boundary temperature of the outer wall surface, i.e. through the indefinite coefficient $B(t)$. Introducing equation (1′) into equation (3′) gives:

$$c_w \rho_w\, \delta\, [\dot{T}_2 - \dot{B}\,(2x^* - x^{*2})] \cong \Lambda \frac{\partial T}{\partial y'}.$$

## 2. SOLUTION OF THE SPECIAL GASO-DYNAMIC PROBLEMS

Proceeding with our solution of the problem we shall use equations (18) and (20) of [1], but we shall modify the boundary conditions for the problem. Starting from the assumption that the pressure of the flowing gas is constant and that both the viscosity coefficient, $\mu$, and the coefficient of thermal conductivity, $\Lambda$, vary linearly with temperature, Chapman and Rubesin showed that the general heat transfer problem can be sub-divided into two particular ones:

(1) the problem of the gas flowing along the wall;
(2) the problem of heat transfer.

The first problem is independent of the second one. The two variables of Blasius being denoted by $\eta$ and $x^*$ in equation (20) of [1] the heat transfer equation will have the following form:

$$\frac{\partial^2 T}{\partial \eta^2} + Pr\, f \frac{\partial T}{\partial \eta} - 2Pr\, f' x^* \frac{\partial T}{\partial x^*} = -\tfrac{1}{4}\, Pr\,(\gamma - 1)\, M^2 (f'')^2.$$

The coefficients of the equation include the generalized Blasius function of flow $f(\eta)$ by which the velocity of the flow $2U^* = f'(\eta)$ is expressed. The equation is to be integrated for the following improved boundary conditions: for the shock wave

$$\eta = \eta_2,\ T = T_2.$$

The general solution for the heat transfer equation is given by the sum:

$$T = T_{\mathrm{I}} + T_{\mathrm{II}}.$$

The particular solution $T_{\mathrm{I}}$ of the equation is a function of $\eta$ and $t$ given as a parameter. The general solution $T_{\mathrm{II}}$ of the homogeneous equation is a function of the variables $\eta$, $x^*$ and $t$ expressed as a parameter.

The particular solution $T_{\mathrm{I}} = T_{\mathrm{I}}(\eta)$ is derived from assumptions which are different from those made in [1]:

$$T_{\mathrm{I}}|_{\eta=0} = T_w|_0$$

$T_w|_0$ is the wall temperature at the initial moment of the flow. This condition is introduced, as opposed to Chapman and Rubesin for it is of special interest to find out about the heating of the wall during the first moments of flow in problems relating to aerodynamic heating of wall.

The particular solution has the form:

$$T_{\mathrm{I}} = T_2 + nT_2\bar{r}(\eta) + c_1\bar{g}(\eta)$$

$$n \equiv \tfrac{1}{2}\,Pr(\gamma - 1)M_2^2$$

$$\bar{r} \equiv \int_{\eta}^{\eta_2}(f'')^{Pr}\int_{\eta_2}^{\xi}(f'')^{2-Pr}\,\mathrm{d}\xi'\mathrm{d}\xi$$

$$\bar{g} = \int_{\eta}^{\eta_2}(f'')^{Pr}\,\mathrm{d}\xi$$

$$c_1\bar{g}(0) \equiv T_w - T_2 - nT_2\bar{r}(0).$$

When the wall temperature is presented by the polynome (1') the general solution of the homogeneous equation has the form:

$$T_{\mathrm{II}} \to (T_2 - T_{w0})Y_0 - B(t)\,(2Y_1x^* - Y_2x^{*2}).$$

In the range of large values of $\eta$ the $Y_k$-functions are expressed through the Chebyshev polynomes $P_{2k}(-i\eta)$ in the following way:

$$y_k \to c_k\,P_{2k}\,(-i\eta)\,.\int_{\eta}^{\eta_2}\frac{e^{-(\eta')^2}}{[P_{2k}\,(-i\eta')]^2}\,\mathrm{d}\eta'.$$

The $Y_0$-function is expressed through the probability integral:

$$Y_0 \to \frac{\phi(\eta_2) - \phi(\eta)}{\phi(\eta_2)},$$

where $\phi(\eta)$ is the probability integral:

$$\phi(\eta) \equiv \int_{\eta}^{\infty} e^{-(\eta')^2}\,\mathrm{d}\eta'.$$

For small values of $\eta$ the $Y_k$-functions were calculated by Chapman and Rubesin (Fig. 5[1]).† With an increase of $\eta$ the $Y_k$-functions rapidly decrease to zero. Thus our assumption (1) can be considered as approximately satisfied. The general solution of the problem of gas heating by a supersonic flow along the walls can, therefore, be presented in the following form:

$$T = T_{\mathrm{I}} + (T_2 - T_{w0})\,Y_0 - B(2Y_1x^* - Y_2x^{*2}) = \theta - B(2Y_1x^* - Y_2x^{*2});$$

$$\theta \equiv T_2\left\{1 + n\bar{r}(\eta) - \frac{\bar{g}(\eta)}{\bar{g}(0)}(1 + n\bar{r}(0)\right\} + T_{w0}\frac{\bar{g}(\eta)}{\bar{g}(0)} + (T_2 - T_{w0})\,Y_0(\eta).$$

As was shown by Chapman and Rubesin (formula 48 of [1]):

$$\Lambda\left.\frac{\partial n}{\partial y}\right|_{y=0} = \frac{\Lambda_2}{2x}\sqrt{(Pe_2)},$$

where: $Pe_2 \equiv \left.\frac{c_p\rho ux}{\Lambda}\right|_2$ is the Peclet number.

Hence we get:

$$c_w\rho_w\,\delta\,[\dot{T}_2 - \dot{B}(2x^* - x^{*2})] = \frac{\Lambda_2}{2x}\sqrt{(Pe_2)}\,[\theta' - B(2Y_1'x^* - Y_2'x^{*2})]|_{y=0}. \quad (4)$$

### 3. DETERMINATION OF THE HEATING OF THE WALL: EXPRESSION OF THE COEFFICIENT $B(t)$.

Thus, the usual differential equation relative to the sought for coefficient $B(t)$, was obtained.

Averaging both parts of equation (4) along the wall we get:

$$c_p\rho_w\,\delta\,(\dot{T}_2 - \tfrac{2}{3}\,\dot{B}) = \tfrac{1}{2}\frac{\Lambda_2}{L}\sqrt{(\bar{P}e_2)}\,[2\theta' - B\,(\tfrac{4}{3}\,Y_{10}' - \tfrac{2}{5}\,Y_{20}')].$$

Using the values of $Y'|_0$ given in Table 1 of [1], namely:

$$Y_0'(0) = -0{\cdot}5915$$
$$Y_1'(0) = -0{\cdot}9775$$
$$Y_2'(0) = -1{\cdot}1949$$

we get:

$$\dot{B} + 0{\cdot}62\Lambda_2\frac{\sqrt{(\bar{P}e_2)}}{c_p\rho_w\delta L}B = -1{\cdot}5\Lambda_2\frac{\sqrt{(\bar{P}e_2)}}{c_p\rho_w\delta L}\,.\,\theta'(0, t) + 1{\cdot}5\dot{T}_2$$

or

$$\dot{B} + 0{\cdot}62m\,B = -\,1{\cdot}5m\,\theta'(0, t) + 1{\cdot}5\dot{T}_2,$$

$$m \equiv \Lambda_2\frac{\sqrt{(\bar{P}e_2)}}{c_p\rho_w\delta L}.$$

† The functions $Y_k$ differ somewhat from the similar functions [1]. but are very near to them.

Let us expand the expression of the derivative:

$$\left.\frac{\partial T_{\mathrm{I}}}{\partial \eta}\right|_0 = nT_2\bar{r}'(0) - T_2\frac{1 + n\bar{r}(0)}{\bar{g}(0)}\bar{g}'(0) + $$
$$+ T_{w0}\frac{\bar{g}'(0)}{\bar{g}(0)} = nT_2\bar{r}'(0) -$$
$$- \frac{\bar{g}'(0)}{\bar{g}(0)}[T_2(1 + n\bar{r}(0) - T_{w0}].$$

$$\left.\frac{\partial \bar{r}(0)}{\partial \eta}\right|_0 = 0{\cdot}603\,(T_2 - T_0) + 3{\cdot}49nT_2;$$

$$n = \tfrac{1}{4}\,Pr(\gamma - 1)\,M_{02}^2 \cong 0{\cdot}072\,M_{02}^2.$$

Hence:

$$\theta'(0, t) = 1{\cdot}234\,[0{\cdot}204\,M_{02}^2 T_2 -$$
$$- 0{\cdot}1035\,(T_2 - T_{w0})] \cong 0{\cdot}252\,M_2^2 T_2.$$

Now we can write the following equation for the averaged $B$:

$$\dot{B} + 0{\cdot}62\,mB \cong -0{\cdot}378\,mM^2T^2 + 1{\cdot}5\,\dot{T}_2,$$

or

$$\dot{B} + \bar{m}B \cong k\dot{T}_2 - \bar{T}_2,$$

where

$$\bar{m} = 0{\cdot}62\,m(t), \quad \bar{T}_2 = 0{\cdot}378\,mM_2^2T^2.$$

By integrating we get:

$$B \cong \int^t (\bar{T}_2 - \bar{T}_2)\exp\left(-\int_t^{t''}\bar{m}\mathrm{d}t'\right)\mathrm{d}t'' +$$
$$+ c\exp\left(-\int^t \bar{m}\mathrm{d}t'\right).$$

By satisfying the initial condition we can obtain the arbitrary constant $c$.

$$c \cong B_0\exp\left(\int^0 \bar{m}\mathrm{d}t'\right) -$$
$$\int^0 (k\dot{T}_2 - \bar{T}_2)\exp\left(\int_0^{t''} m\mathrm{d}t'\right)\mathrm{d}t''.$$

Finally we have:

$$B \cong B_0\exp\left(-\int_0^t \bar{m}\mathrm{d}t'\right) +$$
$$+ \int_0^t (k\dot{T}_2 - T_2)\exp\left(-\int_{t''}^t \bar{m}\mathrm{d}t'\right)\mathrm{d}t''. \quad (5)$$

The expression obtained $B(t)$ gives the solution of the problem stated. The coefficient being known we can find the temperature distribution sought for along the wall in the direction of the flow.

## 4. TREATMENT OF THE OBSERVATIONS MADE BY FISCHER AND NORRIS [3]

The wall temperatures of a V-2 rocket were measured by Fischer and Norris during its supersonic climb at the height of 40 km. It is interesting to note that the rocket was not moving uniformly. The course of its flight is given in Figs. 7 and 4 of [3]. For 22 sec the flight was at a subsonic velocity and supersonic afterwards (Table 1). Fig. 2 gives the variation in time of the temperatures measured at various

*Table 1. The parameters of the supersonic rocket flight*

| $t$, sec | | $U_1$ (m/sec) | $H$ (km) | $T_1$ (°$K$) | $C_1^2 \cdot 10^{-4}$ (m/sec)$^2$ | $C_1$ (m/sec) | $M_1 = \frac{U_1}{c_1}$ |
|---|---|---|---|---|---|---|---|
| From the beginning of the flight | From the beginning of the supersonic flight | | | | | | |
| 0 | — | 0 | 0 | 313 | | | |
| 20 | — | 275 | 6 | 260 | 10·5 | 324 | 0·846 |
| 30 | 08 | 500 | 7·5 | 245 | 9·7 | 311 | 1·61 |
| 40 | 18 | 700 | 12·5 | 230 | 9·32 | 205 | 2·29 |
| 50 | 28 | 1020 | 21 | 218 | 8·84 | 297·5 | 3·44 |
| 60 | 38 | 1500 | 33 | 230 | 9·32 | 305 | 4·92 |
| 72 | 50 | 2750 | 47 | 330 | 13·35 | 365 | 7·5 |

*Table 2.*† *The calculation of the attack angles* $\Delta\alpha = \bar{\alpha} - \alpha$

| $t$ (sec) | $H$ (km) | $T$ (°K) | $M_1$ | $\beta$ | $\sin\beta$ | $M_1 = M_1 \sin\beta$ | $\frac{T_2}{T_1}$ | $T_2$ (°K) | $\bar{\alpha}$ | $\Delta\alpha = \bar{\alpha} - \alpha$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 43 | 40 | 255 | 6 | 33° | 0·545 | 3·18 | 3·27 | 834 | | |
| 38 | 33 | 230 | 4·92 | 35° 24′ | 0·579 | 2·85 | 2·51 | 578 | 25° | 2° 30′ |
| 28 | 21 | 218 | 3·45 | 43° 06′ | 0·683 | 2·35 | 1·985 | 433 | 26° | 3° 30′ |
| 18 | 12·5 | 230 | 2·29 | 53° | 0·799 | 1·83 | 1·58 | 368 | 25° | 2° 30′ |
| 08 | 7·5 | 245 | 1·61 | 90° | 1·0 | 1·61 | 1·388 | | | |

† The time is counted off from the beginning of the supersonic rocket flight equal to 22 seconds.

*Table 3.* $\bar{m}$ *and* $B(t)$ *values*

| $t$ (sec) | $T_2$ (°C) | $\Delta_2 \cdot 10^4$ | $U_1$ (m/sec) / $M_1$ | $\beta$ | $U_2$ (m/sec) | $C_{p2}$ | $\rho_2$ | $Pe_2$ | $\sqrt{(Pe)_2}$ | $\bar{m}$ | $B(t)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 08 | 62 | 0·72 | 500 / 1·59 | 76° | 203 | 0·241 | 1·02 | 20·8 | 4·56 | 1·51 | 29·8 |
| 18 | 95 | 0·76 | 700 / 2·29 | 60° 48′ | 421 | 0·244 | 0·92 | 37·2 | 6·10 | 2·13 | 34·8 |
| 28 | 160 | 0·82 | 1020 / 3·44 | 43° 06′ | 784 | 0·245 | 0·78 | 54·75 | 7·40 | 2·79 | 50·6 |
| 38 | 305 | 1·02 | 1500 / 4·92 | 35° 24′ | 1225 | 0·250 | 0·59 | 55·5 | 7·45 | 3·48 | 96·0 |

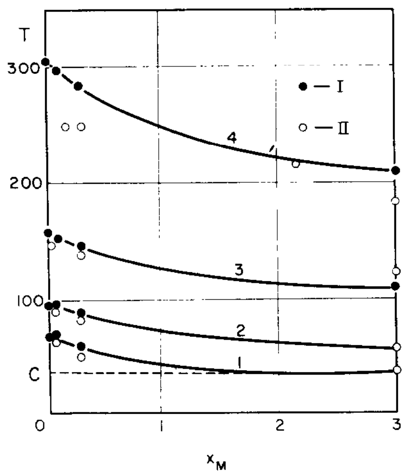


FIG. 2. The course of the temperature variation (°C) at the nose of the rocket at its supersonic flight.
Curve 1: $t =$ 08 sec, $M_1 = 1{\cdot}61$, $T_2 =$ 67°C, $B = 29{\cdot}8$°C;
Curve 2: $t =$ 18 sec, $M_1 = 2{\cdot}29$, $T_2 =$ 95°C, $B = 34{\cdot}8$°C;
Curve 3: $t =$ 28 sec, $M_1 = 3{\cdot}44$. $T_2 =$ 160°C, $B = 50{\cdot}6$°C;
Curve 4: $t =$ 38 sec, $M_1 = 4{\cdot}92$, $T_2 =$ 305°C, $B = 96$ °C.
I—the theory,
II—observations by Fischer and Norris.
The dashed line corresponds to the beginning of the supersonic flight.

points of the surface of the wall which are situated from the nose at distances of 6·33, 15·26, 30·5, 214·2 and 303·1 cm respectively. The temperatures at the nose of the wall are assumed equal to those at the disturbed sides of the oblique shock waves.

The calculations showed that during flight the rocket moved not axially but at some attack angle. The angle ($\alpha$) with the rocket cone half open was, $\alpha = 22° 30'$. The temperatures in the region disturbed by the shock wave were calculated without taking into account the attack angle and turned out to be lower than the temperatures

*Table 4. The temperature distribution along the wall*

| $t$ (sec) | $T_2$ (°C) | | | |
|---|---|---|---|---|
| | $x^* = 0$ | $x = 6{\cdot}3$ | $x = 30$ | $x = 303$ |
| 08 | 67 | 65·8 | 61 | 37 |
| 18 | 95 | 93·5 | 88 | 60 |
| 28 | 160 | 157·9 | 150 | 109 |
| 38 | 305 | 301 | 287 | 209 |

measured.† The attack angles $\Delta\alpha$ of the rocket in flight were calculated (Table 2) according to the temperatures measured near the nose of the rocket at a distance of 6 cm and they turned out to be practically constant.

Table 3 gives calculated values of the coefficients $m$ and $B(t)$ determined as a result of the graphic finding of the solution multipliers (5).

† The theory and calculations were accomplished in 1952, but were not published because of the divergence with the observations indicated above. The corrections for angles of attack were introduced in 1959.

The temperature distributions along the cone wall streamlined for conditions similar to those of the V-2 rocket flight were calculated according to the values obtained. The results of the calculation are given in Table 4 and presented in Fig. 2.

## REFERENCES

1. D. Chapman and M. Rubesin, *J. Aeronaut. Sci.* **16**, No. 9, 547 (1949).
2. A. A. Pomerantsev, *Injenerno-fizicheskii Jurnal* No. 5, (1960).
3. W. Fischer and R. Norris, *Trans. Amer. Soc. Mech. Engrs*, **71**, No. 5, 457 (1949).